Online Casino Games Rules & Strategies

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A Short Introduction

No matter if you're just a freshman to the online gambling industry or you're a real professional, you always need to be aware of the strategies to beat the casino. With the development of the casinos and games, it's no wonder that the strategies are supposed to be modified, as well.

In this book, we're going to come across the most innovative and, evidently, the most effective strategies to augment your winnings at the Canadian online casinos.

We're going to give a particular emphasis to the winning strategies when playing

- Slots
- Roulette, and
- Card games (poker, baccarat, and blackjack included).

So, we're not going to get you bored with plenty of the theory that you'll never be able to apply in real-life gambling—oppositely, we'll equip you with all the necessary practical techniques to calculate the best winning probability!

Here, we'll be providing some outstanding pieces of advice and formulas to ensure you have a higher winning probability. Nevertheless, please kindly keep in mind that **all of these formulas work perfect when the numerous and big bids are made.** To ensure the more accurate results obtained via the formulas, it's necessary to exercise a representative sample based on the numerous big steak bids. The lesser the bid is, the lesser the chance to make these formulas and pieces of advice work in your favor.

In any case, every gambler is to know that the house edge is always higher than the possible winning of a gambler. Nevertheless, our aim is to show you how you can increase your winning within this condition.

In any case, we'll do our best to explain the way you can increase your winning probability by using our brand-new guide provided exclusively to our users! Yes, NativeCasinos is your best advisor in the world of online gambling in Canada!

The first thing you'd know about this slots winning guide is that it's easily applicable to all possible kinds of online slots machines! Yes, no matter if you prefer classic slots or you're into the new slot games, you'll equally be able to make up your new strategy based on our advice!

Secondly, the only deviation for these formulas and tips is the number of reels. Yes, as you may know, there are different numbers of reels of the slots: 3-reel slots, 5-reel slots, etc. Because the number of the reels is different, the calculation of the winning probability is supposed to be conducted in a different manner.

Thirdly, the strategies we're putting down in this guide are also applicable to the following online slot machine types:

- Video slots
- HD slots
- Free spins slots, etc.

So, playing the real money slots will be much easier!

If you wonder, how these strategies, tips and formulas are compatible with the mobile slots, we'll tell you the same: they're absolutely suitable to calculate the winning probability, as well!

Finally for this part, we don't take the bonus slots into account, as it's not significant if a particular slot machine game presupposes any bonuses at all. The sa,e can be said about the progressive jackpot slots and multiplier slots.

Now, let's plunge into the strategies to increase your winnings when playing slots online!

Slots Winning Strategies

To win the slot machines, of course, you need luck—but this is **far from enough**. Behind any winning strategy at the online casinos, the complicated mathematical mechanisms stand.

To fully understand, it is already necessary to refer to the mode of redistribution of bets, it is not linear or random, casinos encourage this idea that everyone thinks: redistribution is random but it is false.

The redistribution is 'conditioned random', that is the redistribution is a function of what the machine has stored, it seems simple, but nothing that the word random implies: periods of redistribution in 'peak' and long periods with small wins here and there, and again we are not talking about jackpot for the moment but simply intermediate wins.[1]

So when you arrive at a slot machine which gives you a little that means that you have arrived at a 'peak', the law of randomness implies that a 'peak' is very rarely alone and very often followed by one to several payout peaks, and this is where all slot machine players fall into the 'trap'.[2]

One, two or three peaks don't mean dozens of peaks in a row. Think otherwise how would the amount of the jackpot win be funded?

"Play Smart" Strategy For Slots

It sounds simple but 98% of slot machine players lose and only 2% cash 100% of the winnings, do you think it's luck?

Certainly not 100% of the time, but rather 20% because of luck (1 in 5) and in 80% of cases playing without a rule is involved.

As a rule, the losers play on a slot machine according to their instincts and the others 2% **winners play strategically**.

To understand the behavior of the winner one must first understand **why the loser does not win**.[3]

Typical case of a loser:

S/he discovers a 'pipe' method on a scam site and immediately s/he opens an account in a casino, pays 100% of his/her gambling capital, plays a little and 10 minutes later he is washed away, the case is settled. You see it is very easy to lose that way.

Now **to make a winner** you have to stick to some strategies including math models application.

First off, pay attention to some general rules to play and win when gambling the slots:

- **Do not play** instinctively but play with a method/strategy

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[1] Dai, L., & Bouguelia, M. R. (2018). Testing exchangeability with martingale for change-point detection. *arXiv preprint arXiv:1810.04022*.
[2] Graydon, C., Dixon, M. J., Gutierrez, J., Stange, M., Larche, C. J., & Kruger, T. B. (2020). Do losses disguised as wins create a "sweet spot" for win overestimates in multiline slots play?. *Addictive Behaviors*, *112*, 106598.
[3] Spetch, M. L., Madan, C. R., Liu, Y. S., & Ludvig, E. A. (2020). Effects of winning cues and relative payout on choice between simulated slot machines. *Addiction*.

- **Do not gamble** at a single casino, this is much too risky you risk losing 100% of the capital, while by having distributed your slot machine capital in 5 or 10 slot machine casinos you do not risk at most 10% to 20% of the capital and above all we fight from the outset against the risk of hanging on to a slot machine
- **Do not will** to make 200% every day before having understood the behavior to be held in relation to each type of slot machine
- **Do not panic**, in the worst case if you make a mistake or encounter a problem only x% of the capital will be played (depending on the number of casinos in which you play, and do not try especially to make up for this loss immediately.

It does not matter because if you have followed our advice on capital dispersion you have lost only 10% of the capital so in truth you have lost at most one day of profit which will be filled the next day. By following the scattering advice you avoid stress and only take pleasure in the slot machines.[4]

In any case, the best solution is to stick to the math models when gambling and be guided by the pieces of advice we mentioned above.

Probability Models To Win Slot Machine Games

The distinct symbols number of the machine $\boldsymbol{S_1}, \boldsymbol{S_2}, \ldots, \boldsymbol{S_p}$ are to be denoted by *p*. If the slot machine gets some blank stops, each of the blanks are supposed to be reviewed as a symbol among them. Therefore, it's paramount to remember that the parameter *p* is specific to the machine.

The payline length is marked by *n*. As a result, *n* is to be considered specific to that payline.

It's also significant to keep in mind that each slot machine can belong to one of the two types. See the table below for more details.

Type A	**Type B**
All reels have the same distribution of symbols	The reels have different numbers of stops and each symbol has different distributions on the stops of the reels

In case A:

- ***t*** is the number of stops on each reel and

[4] Graydon, C., Dixon, M. J., Gutierrez, J., Stange, M., Larche, C. J., & Kruger, T. B. (2020). Do losses disguised as wins create a “sweet spot” for win overestimates in multiline slots play?. *Addictive Behaviors*, *112*, 106598.

- $\boldsymbol{C_i}$ is referred to the distribution (number of instances) of symbol $\boldsymbol{S_i}$ on each reel expressed by the formula ($\mathbf{1} \leq \boldsymbol{i} \leq \boldsymbol{p}$)

In case B,

- $\boldsymbol{t_j}$ is considered as the number of stops on reel number *j* and
- $\boldsymbol{C}_i^j$ is the distribution of symbol $\boldsymbol{S_i}$ on reel number *j* ($\mathbf{1} \leq \boldsymbol{i} \leq \boldsymbol{p}$ and $\mathbf{1} \leq \boldsymbol{j} \leq \boldsymbol{p}$).

Given a specific symbol $\boldsymbol{S_i}$, the probability of $\boldsymbol{S_i}$ emerging on a reel after a spin is $\boldsymbol{q_i} = \frac{Ci}{t}$ in case A and $q_j^j = \frac{Ci}{tj}$ in case B, where *j* is the number of that reel.

Probabilities $\boldsymbol{q_i}$, respectively q_j^j are referred to as *basic probabilities* in slots.

Winning Combinations, Slots Events

As it's known, any winning rule on a payline manifests itself through

- A combination of symbols (for example, the specific combination of 777) or
- A type of symbol combination (for instance, *any bar-symbol twice* or *any triple* of symbols) and any outcome is a specific combination of stops on that line.

As a consequence, the stop combination is supposed to be naturally taken as the probability field's elementary event. As a result, there are $\boldsymbol{t^n}$ available stop combinations in case A and $\boldsymbol{p^n}$ possible symbols' combinations on a payline of length *n* across *n* reels.

In case B, there is the same $\boldsymbol{p^n}$ number of possible symbols' combinations and

$$\prod_{j=1}^{n} \boldsymbol{t_j} = \boldsymbol{t_1 t_2 \ldots t_n}$$

possible stops' combinations for that payline of length *n*.

The events' complexity respectively to the probability computations' ease are manifested in the following manner:

- *Simple events*: A series of the events to be related to one line. These are the types of stops' combinations manifested as specific numbers of identical symbols (instances). For example, on a payline of length 3 this simple event is defined as two seven and one orange symbols.
- *Complex events of type 1*: Related to one line *unions* of simple events. Example: the event *any triple* on a payline of length 3 of a fruit machine is a complex event

of type 1, as being the union of the simple events like *any double* or *two cherries or two oranges* or *at least one cherry* are also complex events of type 1.

- *Complex events of type 2*: The events that are stops' combinations expressed through specific numbers of identical symbols, related to *several* lines. For instance, two seven and one plum symbols on paylines 1, 3, or 5 is a complex event of type 2
- *Complex events of type 3:* The *unions* of events that are types of combinations of stops expressed through specific numbers of identical symbols (similarly to the complex events of type 2), related to *several* lines. For instance, *any triple on paylines 1 or 2* is a complex event of type 3. *At least one cherry on at least one payline* is also a complex event of type 3.

General Formulas Of One Payline The Winning Events' Probability

The general formula of the probability of *E* is as follows:

- In case A:

$$\boldsymbol{P(E)} = \frac{F(E)}{tn} \qquad (1)$$

- In Case B:

$$\boldsymbol{P(E)} = \frac{F(E)}{tn} \qquad (1)$$

Where

- *E* is an event *E* related to a line of length *n*
- *F*(*E*) is the number of combinations of stops favorable for the event *E* to occur.

For an event *E* expressed through the number of instances of *each* symbol on a payline in case A, formula (1) is equivalent to:

$$\boldsymbol{P(E)} = \frac{F(E)}{\prod_{j-1}^{n} tj} \qquad (2)$$

Where $\boldsymbol{P(E)} = \frac{n!}{a1!a2!...ap!} \boldsymbol{(q_1)^{a1}(q_1)^{a2}...(q_1)^{ap}}$ is the number of instances of $\boldsymbol{a_1}$, and so on, $\boldsymbol{S_1}$ is the number of instances of $\boldsymbol{a_p}(\boldsymbol{S_p})$.

It's evident that the second formula can be used for winning events defined through *all* symbols distribution on the payline, in case A. These are to be referred to as the simple events.

The general formula (1) is to be applied for more complex events. This formula reverts to counting the number of favorable combinations of stops $F(E)$, or, for particular situations, apply formula (2) several times and add up the outcomes.

In case B, the number of variables is bigger. As a consequence, most of the explicit formulas from case B are too overloaded. It's necessary to take one particular event for which its probability formula in terms of basic probabilities is presented. Namely, the events are known to be expressed through a number of instances of *one* symbol. If E is the event *exactly m instances of S* ($\boldsymbol{a_1 + a_2 + \ldots + a_p = n}$), then:

$$\boldsymbol{m \leq n} \qquad (3)$$

Where

$$\boldsymbol{P(E) =} \sum_{1 \leq j1 < j2 < \ldots < jn} \prod_{j=\{j1j2\ldots jn\}} q_s^j \prod_{1<n} (1 - q_s^k)$$

and q_s^j are the basic probabilities (the probability of symbol S occurring on reel number j, respectively k).

Several Lines Events' Probability Calculus Tools

It's necessary to call two lines *independent*. That's under a condition if they do not contain stops of the same reel. It means that the result on a line is not related to the outcome of the other and vice versa. As a result, two lines that are not independent will be called *non-independent*.[5]

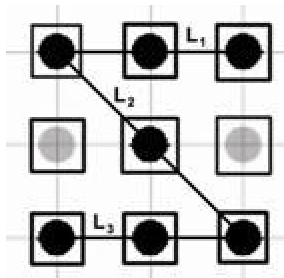
For two non-independent lines, the outcome of one is influenced (partially or totally) by the outcome of the other.

This definition can be extended to several lines (m), as follows.

The m lines are *independent* if every pair of lines from them are independent. Probabilistically, any 2+ events each related to a line from a group of independent lines

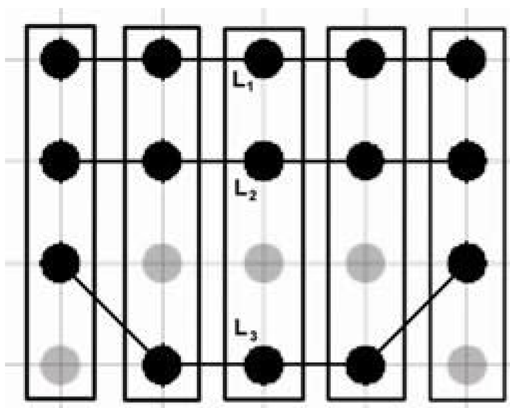
[5] Dai, L., & Bouguelia, M. R. (2018). Testing exchangeability with martingale for change-point detection. *arXiv preprint arXiv:1810.04022*.

are independent, in the sense of the definition of independence of events from probability theory.



Independent and non-independent lines in a 3 x 3-display of a 9-reel slot machine

In the previous figure, lines L_1 and L_1 are independent, while L_1 and L_2, similarly to L_2 and L_1 are non-independent (for the last two pairs, the lines have a stop in common).



Non-independent lines in a 4 x 5-display of a 5-reel slot machine

In the previous figure, lines L_1 and L_2, L_1 and L_1, L_2 and L_1, and therefore L_1, L_2, and L_1 are non-independent. It's because there are stops of the same reel on different lines within each of the mentioned groups. In such a configuration, there is no group of independent lines, regardless the shape or other properties of the lines.

If two lines do not intersect each other, they are not necessarily independent. For instance, take lines L_1 and L_2 in the last figure. On the contrary, lines L_1 and L_2 not intersecting each other in the last but one figure are independent.

The non-independent lines (intersecting or non-intersecting) for which there are non-shared stops belonging to the same reels (similarly to the L_1 **and** L_2 lines) are the *linked lines*. For events related to linked lines, the probability estimations are only possible if we know the *arrangements* of the symbols on the reels, not only their distributions.

Practical Applications and Numerical Probabilities

If the 3-reel slot machines are considered, the following winning events can be exemplified:

E_1	A specific symbol three times
E_2	Any symbol three times (triple)
E_3	A specific symbol exactly twice
E_4	Any symbol exactly twice (double)
E_5	A specific symbol exactly once
E_6	Any combination of two specific symbols
E_7	Any combination of at least one of three specific symbols

If several paylines are touched upon, the following winning events are possible:

- A winning event on any of the horizontal lines
- A winning event on any of the vertical lines
- A winning event on any of the horizontal or vertical lines
- A winning event on either or both of the diagonals
- A winning event on any of the horizontal or diagonal lines
- A winning event on any of the vertical or diagonal lines
- A winning event on any of the horizontal, vertical, or diagonal lines
- A winning event on any of the left-right trapezoidal lines
- A winning event on any of the horizontal or left-right trapezoidal lines

Roulette Winning Strategies

Martingale Roulette Strategy to Win

Martingale is a roulette strategy with more than two hundred years of history and thousands of adherents around the world.

The meaning of the roulette strategy Martingale betting system is that the player, starting the game with the minimum bet, after each loss, doubles it.

Bets must be placed on fields with equal chances of winning. For example, red is black or even-odd. After winning, you must return to the original bet.[6]

Martingale is frequently said to be one of the strategies where you can fail a lot before you win big. To predict the loss, it's possible to use this formula:

$$\sum_{i=1}^{n} B \cdot 2^{i-1} = B(2^n - 1),$$

where the following elements presuppose:

- n is a finite number of bets
- B is the amount of the initial bet.

Besides, it's possible to calculate the probability of not losing:

$$(1 - q^n) \cdot B - q^n \cdot B(2^n - 1) = B(1 - (2q)^n).$$

Herein, q is the probability of losing. For instance, for American double-zero roulette, it is 20/38 for a bet on black or red.

It's evident that the probability of not losing all n bets is $1 - q^n$.

Every time when $q > 1/2$, the expression $1 - (2q)^n < 0$ for all $n > 0$.

Along with the obvious advantages, **this strategy also has disadvantages**. There is a high probability of such an 'endless' increase in the rate, limited only by the player's financial capabilities. It's because according to the fact that the more representative samples are available, the more chances to win.

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[6] Dai, L., & Bouguelia, M. R. (2018). Testing exchangeability with martingale for change-point detection. *arXiv preprint arXiv:1810.04022*.

On Practice

By using the formula mentioned above, we can try to calculate the amount to be won and the amount to be lost:

Martingale is equal to the probability of 6 consecutive losses: $(10/19)^6$ = 2.1256%. Herein, the winning probability can be formulated as 1 minus the probability of 6 time losing:

$$1 - (10/19)^6 = 97.8744\%.$$

As a result, we have the following possible results:

- (1 × 0.978744) = 0.978744 is the expected amount one can win
- (63 × 0.021256) = 1.339118 is the expected amount to be lost
- (0.978744 – 1.339118) = –0.360374 is the total expected value for each application of the betting system.

The best condition to make this strategy work is the unique environment.

There is another type of progressive betting system. It prescribes to do the opposite to Martingale, that is, to reduce the rates by 2 times if you lose and, accordingly, to increase if you win.

There are two categories of bets in roulette: inside and outside. Internal ones are the bets on specific numbers or groups of numbers. Hence, they offer high payouts but low chances of success. Outside bets, for example, on red-black or even-odd, have a high probability of winning, but you will receive a rather modest fee.

Experts recommend paying attention to external rates. And if the payments in case of victory are small, but your chances of success will be more than real. Of course, there is a temptation to put everything on zero and earn a comfortable old age for grandchildren and great-grandchildren.

Yes, huge payouts of 35:1 or 17:1 are great, but the minimal chances of success make it worth it to bet on specific numbers.

Double Up Roulette Strategy

Roulette double up strategy, or roulette doubling strategy, is a ’subdivision’ for the Martingale winning roulette strategy.

Double Up roulette strategy is known to be the easiest strategy ever! Let’s see how it works and how you can benefit from using this approach to beat the casino.

The Double Up roulette strategy concept is quite simple, and these are the actions you're required to do:

- You're supposed to place your bet on one of the very outside bets
- After every coup you have lost, you double your bet, and you keep doing that until you win.

Just two easy steps.

The Labouchère System

The Labouchère system used for roulette betting tends to guarantee 18/38 chance of success betting, which is around 47.37%. So, it's not a 50/50 probability of winning—but even if less.[7]

From a theoretical point of view, in case if the player is cancelling out 2 numbers on the list for every win and adding only 1 number for every loss, a gambler will have his proposition come at least 33.34% to eventually fulfill the list.

$$(y + z) / 2 \leq x,$$

Where

x = Number of wins

y = Losses Number

z = Numbers included originally on the list

Nevertheless, under this condition, there'll be a moment when a gambler is unable to make any bets anymore.

Thus, it's possible to use another formula:

$y + (z / 2) \leq x$, where

x = Number of wins

y = Number of losses

z = Numbers originally on the list.

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[7] Du, M., Sassioui, R., Varisteas, G., Brorsson, M., & Cherkaoui, O. (2017, November). Improving real-time bidding using a constrained markov decision process. In *International Conference on Advanced Data Mining and Applications* (pp. 711-726). Springer, Cham.

The Labouchère strategy can be applied to live roulette, and in some cases even better than real roulette.

The interface allows you to get valuable information about hot and cold numbers, as well as the percentage of winnings for each type of bet, and the number of exact numbers drawn. On average, information is provided for the previous 500 rounds, giving you an accurate idea of how you should play at this table.

Before you start playing roulette, you should prepare yourself, namely, decide on the roulette option, as well as set a time and money limit for yourself. After that, you can choose a strategy and sit down at the gaming table. It will be important to set your stop-win and stop-loss: the amounts at which you will end the game.

An experienced player can always set these goals and follow them. By focusing on the result, you can make the smartest decisions. To become a cold-blooded player, you need to abstract from the euphoria of victory, or the feeling of having to recoup.

On Practice

For example, there's a situation where the list starts with 7 numbers. The player wins 5 times and loses 3 (62.5% winning percentage) the list is completed. In this case, the player wins the amount planned and desired, if the list starts with 7 numbers and the player wins 43,600 times and loses 87,193 times (33.34% winning percentage) the list completes and the player wins.

The Reverse Labouchère Betting System

The Labouchère roulette betting system can also be played in the positive progression betting system format. It's usually referred to as the reverse Labouchère.

In accordance with this approach, the player is supposed to add the previous bet amount to the end of the line instead of deleting numbers from the line. You continue building up your Labouchère line until you hit the table maximum.

After a loss, the player deletes the outside numbers and continues working on the shorter line. The player starts their line again if they run out of numbers to bet.

To determine the moment and the way the system fails, it's necessary to use this formula:

$$x + z \leq y * 2,$$

Where:

x = Number of wins

y = Number of losses

z = Numbers on original list.

In this case, the system is considered to have failed. Thus, all of the numbers on the line are crossed absolutely out.

On Practice

The reverse Labouchère method is applicable to simple bet roulette. The system is based on the fact that initially the player needs to decide on the amount that he wants to receive for the game cycle.

Let's say we want to win C$5000. To build a cycle, let's divide this amount into four different parts: 500, 1000, 1500, 2000:

- The first bet, to start the game according to this strategy, will be the sum of the first and last parts. In our example, this is C$2500 (500 + 2000)
- If we lose, then we add the fifth part (2500) to our list of the cycle
- If we win, then we remove the first and last numbers from the list (500 and 2000)
- If all the numbers from the list are removed, it will mean that we have won the desired amount.

Once you understand a little about the principle of the reverse Labouchère best strategy roulette, you can change the numbers in sequence. For example, if you want to win $5000, you can make a sequence of three parts (1000 - 1000 - 3000) or five (1000 - 1000 - 1000 - 1000 - 1000), in general, as it is convenient for you.

The Andrucci Roulette System

The Andrucci roulette betting system is based on the so-called straight up bets. It means that you're supposed to make a bet on your lucky number only. It's because the payouts for this approach are the highest and are equal to 35:1.

So, how does this roulette odds strategy works:

- You'll need to record the numbers being successful in the game
- You're to stick to the number of spins varying between 15 and 25
- As a rule, the 34 first spins are to bring a win.

Nevertheless, the most important part of the system is observation!

On Practice

Before gambling for real money and using the Andrucci system, it's important to know that

- The numbers that are the most frequent to occur are 1, 8, and 15
- By wagering, say, C$10 on 15 black for 8 times, and lose $80
- With the ninth round, you'll bet the win 35:1 that is C$350.

If you lost C$80, you have C$270 pure winning! Remember that your initial bet was just C$10.

The D'Alembert System

The main idea of the D'Alembert system is that after a loss, the player must bet more than s/he lost on the previous bet.

Applying this system to roulette (even odds bets), the player must increase the bet after each loss, and decrease the bet after winning. The table shows the possible results of 11 bets, where, as a result, the player will have C$6 more.

As a shortcoming of the game, there is a likelihood of a prolonged series of failures (when, for example, black comes up much more often than red), as a result of which you will lose your play capital or reach the upper betting limit.

On Practice

Practically, the winning are possible to be distributed in the following manner:

Game Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Your bet	C$1	C$2	C$3	C$2	C$3	C$2	C$1	C$2	C$1	C$2	C$1
Previous Bet +/- 1 chip	N/A	C$1 + C$1	C$2 + C$1	C$3 - C$1	C$2 + C$1	C$3 - C$1	C$2 - C$1	C$1 +C$1	C$2 - C$1	C$1 + C$1	C$2 - C$1
Red or Black	Black	Black	Red	Black	Red	Red	Black	Red	Black	Red	Red
Your win	-$1	-$2	+$3	-$2	+$3	+$2	-$1	+$2	-$1	+$2	+$1
Total win	-$1	-$3	+$0	-$2	+$1	+$3	+$2	+$4	+$3	+$5	+$6

So, by utilizing the bets amounts in compliance with the game number, you'll be able to increase your winning probability.

The Fibonacci Betting System

This is one of the roulette strategies that work. This system is based on the even numbers bets. Also, the key principle of the betting system is based on Fibonacci numbers sequence. By using this theory you'll be able to have a 50% probability of winning!

Let's consider how it works practically!

On Practice

The numbers sequence by Fibonacci is as follows:

1 – 1 – 2 – 3 – 5 – 8 – 13 – 21 – 34 – 55 – 89 – 144 – 233 – 377 – 610 – 987

Before you move to the next number, it's necessary to await the first win. It means that in case of the first win, you're supposed to increase by one.

The Paroli Betting System

The Paroli roulette strategy to win at the casino is designed for betting on equal chances including

- Red/black
- Small/large, and
- Even/odd.

This system belongs to the category of positive progressions.

The basic principle of the Paroli strategy is very simple to describe:

- After each win, the player is instructed to double the size of the bet
- After each loss it's necessary to stop the sequence and start from the very beginning, with a bet of 1 unit.

No complicated mathematical operations, no addition and subtraction—nothing like that! That is why the Paroli betting system is almost as popular with novice roulette players as the legendary Martingale strategy.

On Practice

On practice, the main advantage of the Paroli strategy is that even in the case of a long series of losses, the player does not have any problems—each time he simply makes a minimum bet of 1 unit.[8]

Thus, unlike the same Martingale system, there is no risk of spending the entire bankroll and not getting the opportunity to recoup due to table limits. Bets only go up if you win, which means that even after meeting the table limits, you will already be in the black.[9]

In addition, a consistent series of wins through the use of the Passwords betting system can provide a player with very large profits.

1 3 2 6 Casino Roulette Strategy

When playing using the 1 3 2 6 online roulette strategies, you will be exposed to risk losing 2 chips, and you can win 12. So, it is some maximum profit with minimum investment when applying this roulette gambling strategy.

1 3 2 6 winning strategy for roulette is good for

- Positive progression bets (where you're to increase if you win) and
- Simple odds play (Red–Black; Over–Under; Even–Odd).

On Practice

When using this strategy, your cash back is 2 chips.

The betting progression should be 1-3-2-6. When you win a bet, the next one is made in progression. If the bet is lost, we start betting again from the initial bet.

As for the completion, you'll have the following:

- Losing 2 chips or
- Winning 12 chips.

Practically, it works like that. If the first bet is lost, 1 chip is lost. If the second bet is lost, 2 chips are lost. If the third bet is lost, 2 chips are won. If you lose 5 out of 6 sessions on the second bet, and win 4 bets in a row in one session, you win 2 chips.

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[8] Han, Y., & Wang, G. (2018). Expectation of the Largest Betting Size in Labouchère System. *CoRR*.

[9] Du, M., Sassioui, R., Varisteas, G., Brorsson, M., & Cherkaoui, O. (2017, November). Improving real-time bidding using a constrained markov decision process. In *International Conference on Advanced Data Mining and Applications* (pp. 711-726). Springer, Cham.

Card Games Winning Strategies

Understanding the mathematics of a game also is important for the casino operator to ensure that the reasonable expectations of the players are met. For most persons, gambling is entertainment. It provides an outlet for adult play. As such, persons have the opportunity for a pleasant diversion from ordinary life and from societal and personal pressures.

As an entertainment alternative, however, players may consider the value of the gambling experience. For example, some people may have the option of either spending a hundred dollars during an evening by going to a professional basketball game or at a licensed casino. If the house advantage is too strong and the person loses his/her money too quickly, s/he may not value that casino entertainment experience. On the other hand, if a casino can entertain him/her for an evening, and s/he enjoys a 'complimentary' meal or drinks, s/he may want to repeat the experience, even over a professional basketball game.

Likewise, new casino games themselves may succeed or fail based on player expectations. In recent years, casinos have debuted a variety of new games that attempt to garner player interest and keep their attention. Regardless of whether a game is fun or interesting to play, most often a player will not want to play games where his/her money is lost too quickly or where s/he has an exceptionally remote chance of returning home with winnings.

Poker & Video Poker

Martingale Betting System

Instead of pouring tons of theory, we'd better explain this betting system by an example. For example, you have a nice sum of C$2, 550, 000 = $(2^8 - 1) \cdot 10, 000$). You decide to play at a casino and devise the following gambling scheme. Every month you go to the poker table and bet C$10, 000. If you win, you just won C$10, 000 and you quit and live off your money for a month.

Now if you lose, you decide to play again, and you double your bet to C$20, 000. If you win your second bet then your net win is

C$20, 000 – C$10, 000 = C$10, 000.

Again if you lose you double you bet, etc.

The martingale betting system consists then of doubling your bet until your first win. In any case if you win at your k^{th} bet then your net gain is to be calculated in the following manner:

$$10000\,[-1-2-2^{k-1}+2^k] = 10000$$

using the geometric series $1 + x + \cdots + x^{k1} = \frac{1-Xk}{1-X}$ with $x = 2$.

So with the martingale betting system, you do win 10000 every time.

If you have unlimited resources (and if the casino has no betting limit) you could in principle make money using subfair games. But of course none of this condition is true. Suppose that like in our example you can bet at most n bets in a row before running out of money. If the probability to lose any single game is q then

$$\text{Probability to lose everything} = q^n$$

since to lose everything you need to lose n times in a row. Let us compute the expected gain W playing the game this way. We have

$$E[W] = 10000 \cdot (1 - q^n) - (2^n - 1)10000 \cdot q^n = 10000\,[1 - (2q)^n]$$

If $n = 8$ and the game were fair $q = 1/2$ then the probability to lose everything on a single month is 1/256 = 0.0039 and the expected gain is 0. If you play poker for which $q = 251/295$ then the probability to lose everything is 0.0044 and the expected gain is −\$1188.92 which is about 11% of your bet size, which is not very good. To compare various n note that for poker

n	6	7	8	9	10
E[W]/10000	-0.08	-0.10	-0.11	-0.13	-0.15

so actually the more money you have to play the martingale strategy, the more you lose on average.

The idea behind this betting (and many other) betting systems is to make sure that you win (a little) with high probability and to make you forget that when you eventually lose, you do lose a lot. To analyze the martingale betting a bit better, let us compute how many times, on average, should one play to see n loss in a row. We shall do this using a first order difference equation. We define

$$x(n) = \text{Expected number of games until you lose n times in a row}$$

Let us start with $x(1)$: if the first game is a loss (with probability q) then $x(1) = 1$, while of the first game is a win(with probability p), then the expected number of games until 1 loss will be 1 + $x(1)$ so that we have the equation $x(1) = 1 \times q + (x(1) + 1) \times p$. which is easily solved to give $x(1) = \frac{1}{q}$. For $x(n)$ let us concentrate on what happens after we have sustained $n - 1$ loss in a row (this took $x(n-1)$ games on average). If we lose (with probability q) then we need $x(n-1) + 1$ games to lose n in a row. But if we win that game, we start afresh and it will take then $x(n-1) + 1 + x(n)$ games to reach n losses. That is we have the equation

$$x(n) = [x(n-1) + 1] \times q + [x(n-1) + 1 + x(n)]p$$

which gives the difference equation

$$x(n) = \frac{1}{q}\, x(n-1) + \frac{1}{q}$$

with the initial condition $x(1) = 1/q$. We find the solution by using the following formula:

$$x(n) = \frac{1}{p}\left[\left(\frac{1}{q}\right)\right]^{n} - 1 \;.$$

For poker we get for example

n	6	7	8	9	10
$x(n)$	117.3	233	462	913	1803

So, for example, if you play at roulette 500 hundred times, you should expect to loose 8 times in a row! In the martingale betting system during a single game at the casino, you play until you win once, so it takes on average $1/p$ games to achieve that. As a result, on average you will visit the casino $(1/q)^n - 1$ times (227 times with n=8) before you go bust. **You could as well spend C$10,000 every month and your money will last longer, namely 255 weeks.**[10]

Let's have a look at some more practical examples of applying martingale betting strategy:

- Your initial stake is C$1
- Bet of C$1, either you win C$2 minus your previous bet 2–1 = C$1, or you lose
- Bet of C$2, either you win C$4 less your previous bet: 4–2–1 = C$1, or you lose
- Bet of C$4, either you win C$8 minus your previous bet 8–4–2–1 = C$1, or you lose

[10] Kendall, G. (2018). Did a roulette system "break the bank"?. *Significance*, *15*(6), 26-29.

- Bet of C$8, either you win C$16 less your previous bet 16–8–4–2–1 = C$1, or you lose
- Bet of C$16, either you win C$32 minus your previous bet 32–16–8–4–2–1 = C$1, or you lose
- Bet of C$32, either you win C$64 less your previous bet 64–32–16–8–4–2–1 = C$1, or you lose
- Bet of C$64, either you win C$128 minus your previous bet 128–64–32–16–8–4–2–1 = C$1, or you lose
- Bet of C$128 euros, either you win C$256 minus your previous bet 256–128–64–32–16–8–4–2–1 = C$1, or you lose.

In short, the more the player loses, the more he must bet to win just C$1 euro.

The wheels have a '0' which is neither red nor black, not even or odd, not missing or passed. The elementary probability of winning on each draw for a bet of the red, black, even, odd, miss or pass type is therefore 18/37 (0.48648) and not 1/2. The probability of winning by having an infinite sum tends towards 1 in terms of mathematical limit.

In addition, to paralyze this strategy, casinos offer game tables per wager: from C$1 to C$100, from 2 to 200, from 5 to 500, etc.[11] It is therefore impossible to use this method over a large number of shots, which increases the risk of losing everything, without being able to redo.

Baccarat

First of all, we would like to calculate the probabilities of win, lose and tie for Player. Let us define several random variables.

N: total number of plays

X_w: total number of wins for Player

X_l: total number of loses for Player

X_t: total number of Ties

p_w: probability of win for Player

p_l: probability of lose for Player

p_t : probability of Tie

[11] Kendall, G. (2018). Did a roulette system "break the bank"?. *Significance*, *15*(6), 26-29.

where $p_w, p_l, p_t \in (0, 1)$, $N = X_w + X_l + X_t$, and $p_w + p_l + p_t = 1$. So random variables X_w, X_l, X_t follow a multinomial distribution.

$$(X_w, X_l, X_t) \sim \text{Multi}(N, p_w, p_l, p_t).$$

The unbiased estimator of p_w is

$$\widehat{p}_w = \frac{Xw}{N}.$$

Variance of $\widehat{p}_w$ is

$$\text{Var}(\widehat{p}_w) = \frac{Var(Xw)}{N^2} = \frac{p_w(1 - p_w)}{N}.$$

Because p_w is unknown, so we use $\widehat{p}_w$ instead, $\text{Var}(\widehat{p}_w) = \widehat{p}_w(1 - \widehat{p}_w)/\text{N}$.

Likewise, $\widehat{p}_l = X_l/N$, $\text{Var}(\widehat{p}_l) = \widehat{p}_l(1 - \widehat{p}_l)/\text{N}$, and $\widehat{p}_l = X_t/N$, $\text{Var}(\widehat{p}_t) = \widehat{p}_t(1 - \widehat{p}_t)/N$.

1000 rounds of game are simulated, it is not a difficult work to calculate the estimated pw, pl , pt values and its standard errors.

The results mean that in one play the probability of win for Player is about 0.44539098, the probability of loss for Player is about 0.45932556, the probability of tie is about 0.09528346.

When betting on Player or Banker, the 'Tie' outcomes will not earn or lose you any money, so we can ignore the 'Tie' outcomes in this situation.[12]

The simulation results show that if bet on Player, the average probability of win is about 0.4922989, the standard error is 0.0006114399, 95% CI is: (0.4911005, 0.4934973). The average probability of loss is about 0.5077011, the standard error is 0.0006114399, 95% CI is: (0.5065027, 0.5088995). The average probability of Tie is 0.09528346, the standard error is 0.0003415518, 95% CI is: (0.09461402, 0.0959529).

Baccarat Expected Return Per Player

Using the estimated probabilities in the previous section, we can calculate the expected return per Play (ERP) easily. Here we assume the bet on each play is 1.

If bet on Player, the ERP is calculated as

[12] Rute, J. (2016). Computable randomness and betting for computable probability spaces. *Mathematical Logic Quarterly*, *62*(4-5), 335-366.

$$\text{ERP} = 0.4922989 - 0.5077011 = -0.0154022.$$

If bet on Banker

- For commission Baccarat the ERP is calculated as

$$\text{ERP} = 0.95 \times 0.5077011 - 0.4922989 = -0.009982855.$$

- For no commission baccarat, the ERP is about −0.0144231 (calculated through simulation).

If bet on Tie, the ERP is calculated as

$$\text{ERP} = 0.09528346 \times 9 - 1 = -0.1424489.$$

We can note, all the expected incomes are negative which means the gambler is expected to lose money on the average. Yes, the casino is always winning. However, if there's a chance to lose, it means that there's always a chance to win! That's the best aspect about any gambling game![13]

One interesting thing of note from a psychological aspect is that people would like to play no commission baccarat; however, in fact commission baccarat would have a higher ERP.

Blackjack

Markov Chain For Blackjack

Markov chains are an important class of random walks in that they have finite memory: knowing the current state provides as much information about the future as knowing the entire past history. Defining a particular Markov chain requires a *state space* (the collection of possible states) and a *transition matrix*.[14] The entry in row i and column j of the transition matrix is the probability of moving from state i to state j in one step.

States that only allow transitions back to themselves (with probability 1) are called absorbing states. If we raise the transition matrix to the n^{th} power, entry (i, j) is the probability of moving from state i to state j in exactly n steps. Certain classes of Markov

[13] Körmendi, A., Csinády, A., Kurucz, G., & Balázs, D. (2018). Close-to-win evaluations are affected by the outcome and delay between stopping the wheels in slot machines. *Psychiatria Hungarica: A Magyar Pszichiatriai Tarsasag tudomanyos folyoirata*, *33*(4), 340.

[14] Du, M., Sassioui, R., Varisteas, G., Brorsson, M., & Cherkaoui, O. (2017, November). Improving real-time bidding using a constrained markov decision process. In *International Conference on Advanced Data Mining and Applications* (pp. 711-726). Springer, Cham.

chains will converge to an equilibrium distribution as n gets large. This equilibrium represents the long-term proportion of time that the chain spends in each state (independent of the starting state).

Markov chains which do converge to equilibrium are those that are irreducible and aperiodic. A chain is irreducible if there exists a path of non-zero probability from every state to every other state. If a chain is irreducible, it is also aperiodic if when considering all possible paths from any state *i* back to itself, the greatest common denominator of the path lengths is one.

In the card-counting technique of the Complete Point-Count System within Markov chains system, all cards in the deck are classified as

- low (2 through 6)
- medium (7 through 9), or
- high (10 and Ace).

Each 52-card deck thus contains

- 20 low cards,
- 12 medium cards, and
- 20 high cards.

As the round progresses, the player keeps track of an ordered triple (L, M, H), representing the number of low, medium and high cards that have been played. This triple is sufficient to compute the number of cards remaining in the shoe, $R = N - (L + M + H)$, where $N = 52\Delta$ is the number of total cards in the shoe. The player uses the ordered triple to compute a *high-low index* (HLI):

$$\text{HLI} = 100 \cdot \frac{L - H}{R}$$

The HLI gives an estimate of the condition of the shoe: when positive, the player generally has an advantage and should bet high; when negative, the player generally has a disadvantage and should bet low. So, it is possible to offer one possible suggestion for varying bets:

$$B = b \qquad \text{if } -100 \le \text{HLI} \le 2,$$

$$B = \left[\frac{HLI}{2}\right] b \quad \text{if } 2 < \text{HLI} \le 10,$$

$$B = 5b \qquad \text{if } 10 < \text{HLI} \le 100,$$

where b is the player's fundamental unit bet. It is important also to note that, although the player's advantage is presumably high when HLI > 10, it's possible to recommend an upper limit on bets for practical reasons. If a casino suspects that a player is counting cards, they will often remove that player from the game.[15] Finally, it's possible to also recommend taking the insurance bet when HLI > 8.

Suppose that the player observes an ordered triple (L, M, H) prior to beginning a hand. From the triple we may compute the HLI and obtain the strategy S.

First, we assume that the distribution ☐ is uniform over each category:

- low,
- medium, and
- high.

To be precise, we set ☐ as follows:

$$d(2) = \ldots = d(6) = \left(\tfrac{1}{5}\right) \frac{20\Delta - L}{R} ,$$

$$d(7) = d(8) = d(9) = \left(\tfrac{1}{3}\right) \frac{12\Delta - M}{R} ,$$

$$d(10) = \left(\tfrac{4}{5}\right) \frac{20\Delta - H}{R} ,$$

$$d(\mathrm{A}) = \left(\tfrac{1}{5}\right) \frac{20\Delta - H}{R} .$$

Second, we assume that ☐ does not change during the play of the hand. Third, we require that the player fixes a strategy at the beginning of the hand (based on the HLI) — that is, the player does not respond to changes in the HLI until the hand is complete.

With these assumptions, we are able to compute the player's advantage for a hand beginning with any arbitrary triple (L, M, H). If we were also able to determine the overall probability that the player begins a hand with triple (L, M, H), we would be able to compute the overall long-term advantage simply by averaging over all triples. We turn once again to Markov chains to find these probabilities.

As a result, the overall expected gain for player using the Complete Point-Count System is as follows,

Δ	**Expected gain, *g***
1	0.0296

[15] Moffitt, S. (2017). Gambling for quants, part 1: A simple fractional betting system.

2	0.0129
4	0.0030

It's evident that in comparison to baccarat winning probability, when playing blackjack, there's a bigger opportunity to win from a casino. However, this probability is not as big as it was anticipated.

We have presented an analysis framework for card-counting systems that operates completely without simulation. This framework is based on the observation that certain aspects of the game of blackjack have finite memory and are well modeled by discrete Markov chains.

By using techniques from Markov chain analysis, we developed a framework for calculating the player's long-term expected gain and exercised this technique on the well-known Complete Point-Count System. By using our method, one could further investigate detailed aspects of the game. For example, by determining the reasons underlying particular advantages, a player could assess rule variations and make strategy adjustments accordingly.

Math Behind Blackjack Winning Probability

Suppose two card decks are in play, then by using the combinatorial function, the player could determine that there are eight Aces and thirty-two 10 valued cards available to create a 21 as well as there being 104 total cards in the deck. With this information, the function is used by stating: there are 8 Aces with only one needing to be obtained multiplied by 32 10-valued cards with only one needing to be obtained all divided by 104 total cards with only two needing to be obtained to create the pair.

The terminology is portrayed as "8 choose 1 multiplied by 32 choose 1, all divided by 104 choose 2."[16] This specific calculation gives a value of a 4.78% chance the player receives 21 on the first two cards she receives. That is, if we let J, Q, and K denote face cards Jack, Queen, and King, we have:

- In the Case of 2 Decks:

$$P(\{A, 10\} \text{ or } \{A, J\} \text{ or } \{A, Q\} \text{ or } \{A, K\}) = \frac{\binom{8}{1}\binom{32}{1}}{\binom{104}{2}} = 4{,}78\%.$$

- In the Case of 3 Decks:

[16] Cooke, C. H. (2010). Probability models for blackjack poker. *Computers & mathematics with applications*, *59*(1), 108-114.

$$P(\{A, 10\} \text{ or } \{A, J\} \text{ or } \{A, Q\} \text{ or } \{A, K\}) = \frac{\binom{12}{1}\binom{48}{1}}{\binom{156}{2}} = 4.76\%$$

- In the Case of 4 Decks:

$$P(\{A, 10\} \text{ or } \{A, J\} \text{ or } \{A, Q\} \text{ or } \{A, K\}) = \frac{\binom{16}{1}\binom{64}{1}}{\binom{208}{2}} = 4.76\%$$

- In the Case of 5 Decks:

$$P(\{A, 10\} \text{ or } \{A, J\} \text{ or } \{A, Q\} \text{ or } \{A, K\}) = \frac{\binom{20}{1}\binom{80}{1}}{\binom{260}{2}} = 4.75\%$$

- In the Case of 6 Decks:

$$P(\{A, 10\} \text{ or } \{A, J\} \text{ or } \{A, Q\} \text{ or } \{A, K\}) = \frac{\binom{24}{1}\binom{96}{1}}{\binom{104}{2}} = 4.75\%$$ [17]

A Third Card

Suppose two card decks are still in play, then by using ratios, the player could determine whether to hit or stand based on her best and worst chances of receiving a third card without exceeding 21. For example, consider the player has a pair of two 10-valued cards (e.g. {10, 10}, {10, J}, {Q, K}, etc.) and would need only one of the possible eight Aces to not exceed 21. The scenario would proceed as follows: after each player and the dealer has their pair of cards, the deck has 96 cards remaining.

The "Best Case Scenario" is all 8 Aces are still in the remaining deck of cards because no other player has any Aces among the eight cards in their hands combined. Thus, the ratio of 8 Aces to 96 cards shows my chances of receiving an Ace.

For the "Worst Case Scenario", all other players have all of the Aces in their hands already, leaving two Aces in the deck. Only two are left because the player is aware of the eight possible Aces in the deck with the possibility of six being taken because she knows she does not have Aces in her own hand. This gives a ratio of 2 Aces to 96 cards chance of not exceeding 21. These scenarios create an interval of success for the player to

[17] Cooke, C. H. (2010). Probability models for blackjack poker. *Computers & mathematics with applications*, *59*(1), 108-114.

remain in the game. In this case the player has a 2.08% - 8.33% chance of success in receiving an Ace.

The Worst and Best Case scenarios are taking the two extremes of trying to not exceed 21, thus giving an interval for all other possible situations (e.g. only 3 Ace through 7 Aces in the remaining deck). Calculations are given for card pairs totaling at 20 with each card equaling 10 ({10, 10}, {10, J}, {10, Q}, {10, K}, {J, Q}, {J, K}, {Q, K}) for decks two through six:

- In the Case of 2 Decks:

Best Case: All 8 Aces Are in the Deck of 96 Cards

$$8 \text{ Aces} : 96 \text{ Cards} = \frac{8}{96} = 8.33\%$$

Worst Case: Only 2 Aces Are in the Deck of 96 Cards

$$2 \text{ Aces} : 96 \text{ Cards} = \frac{2}{96} = 2.08\%$$

Interval: 2.08% – 8.33%

- In the Case of 3 Decks:

Best Case: All 12 Aces Are in the Deck of 148 Cards

$$12 \text{ Aces} : 148 \text{ Cards} = \frac{12}{148} = 8.11\%$$

Worst Case: Only 6 Aces Are in the Deck of 148 Cards

$$6 \text{ Aces} : 148 \text{ Cards} = \frac{6}{148} = 4.05\%$$

Interval: 4.05% – 8.11%

- In the Case of 4 Decks:

Best Case: All 16 Aces Are in the Deck of 200 Cards

$$16 \text{ Aces} : 200 \text{ Cards} = \frac{16}{200} = 8.00\%$$

Worst Case: Only 10 Aces Are in the Deck of 200 Cards

$$10 \text{ Aces} : 200 \text{ Cards} = \frac{10}{200} = 5.00\%$$

Interval: 5.00% – 8.00%

- In the Case of 5 Decks:

Best Case: All 20 Aces Are in the Deck of 252 Cards

$$20 \text{ Aces} : 252 \text{ Cards} = \frac{20}{252} = 7.94\%$$

Worst Case: Only 14 Aces Are in the Deck of 252 Cards

$$14 \text{ Aces} : 252 \text{ Cards} = \frac{14}{252} = 5.56\%$$

Interval: 5.56% – 7.94%

- In the Case of 6 Decks:

Best Case: All 24 Aces Are in the Deck of 304 Cards

$$24 \text{ Aces} : 304 \text{ Cards} = \frac{24}{304} = 7.89\%$$

Worst Case: Only 18 Aces Are in the Deck of 304 Cards

$$18 \text{ Aces} : 304 \text{ Cards} = \frac{18}{304} = 5.92\%$$ [18]

Interval: 5.92% – 7.89%.

Thus, the probabilities can be calculated by taking into account four essential cases.

Case 1 Probability

Case 1 contains the pairs of the same card with point values not equal to 10 (e.g. {A, A}, {2, 2}, {3, 3}, {4, 4}, {5, 5}, {6, 6}, {7, 7}, {8, 8}, and {9, 9}). Thus, the probabilities are as follows:

Number of decks	Calculation	Probability
2	$\frac{\binom{8}{2}}{\binom{104}{2}}$	0.52%

[18] Cooke, C. H. (2010). Probability models for blackjack poker. *Computers & mathematics with applications*, *59*(1), 108-114.

3	$\frac{\binom{12}{2}}{\binom{156}{2}}$	0.55%
4	$\frac{\binom{16}{2}}{\binom{208}{2}}$	0.56%
5	$\frac{\binom{20}{2}}{\binom{260}{2}}$	0.56%
6	$\frac{\binom{24}{2}}{\binom{312}{2}}$	0.57%

Case 2 Probability

Case 2 contains the pairs where exactly one card is valued at 10 (e.g. {A, 10 or J or Q or K}, {2, 10 or J or Q or K}, {3, 10 or J or Q or K}, {4, 10 or J or Q or K}, {5, 10 or J or Q or K}, {6, 10 or J or Q or K}, {7, 10 or J or Q or K}, {8, 10 or J or Q or K}, {9, 10 or J or Q or K}.

Number of decks	Calculation	Probability
2	$\frac{\binom{8}{1}\binom{32}{1}}{\binom{104}{2}}$	4.78%
3	$\frac{\binom{16}{1}\binom{64}{1}}{\binom{208}{2}}$	4.76%
4	$\frac{\binom{16}{1}\binom{64}{1}}{\binom{208}{2}}$	4.76%

5	$\frac{\binom{20}{1}\binom{80}{1}}{\binom{260}{2}}$	4.75%
6	$\frac{\binom{24}{1}\binom{96}{1}}{\binom{104}{2}}$	4.75%

Case 3 Probability

Case 3 contains the pairs where both cards value at 10 (e.g. {10, 10}, {10, J}, {10, Q}, {10, K}, {J, Q}, {J, K}, and {Q, K}).

Number of decks	**Calculation**	**Probability**
2	$\frac{\binom{32}{2}}{\binom{104}{2}}$	9.26%
3	$\frac{\binom{48}{2}}{\binom{156}{2}}$	9.33%
4	$\frac{\binom{64}{2}}{\binom{208}{2}}$	9.36%
5	$\frac{\binom{80}{2}}{\binom{260}{2}}$	9.39%
6	$\frac{\binom{96}{2}}{\binom{312}{2}}$	9.40%

Case 4 Probability

Case 4 contains all of the other pairs possible in the deck (e.g. {A, 2 or 3 or 4 or 5 or 6 or 7 or 8 or 9}, {2, 3 or 4 or 5 or 6 or 7 or 8 or 9}, {3, 4 or 5 or 6 or 7 or 8 or 9}, {4, 5 or 6 or 7 or 8 or 9}, {5, 6 or 7 or 8 or 9}, {6, 7 or 8 or 9}, {7, 8 or 9}, and {8, 9}).[19]

Number of decks	Calculation	Probability
2	$\frac{\binom{8}{1}\binom{32}{1}}{\binom{104}{2}}$	4.78%
3	$\frac{\binom{16}{1}\binom{64}{1}}{\binom{208}{2}}$	4.76%
4	$\frac{\binom{16}{1}\binom{64}{1}}{\binom{208}{2}}$	4.76%
5	$\frac{\binom{20}{1}\binom{80}{1}}{\binom{260}{2}}$	4.75%
6	$\frac{\binom{24}{1}\binom{96}{1}}{\binom{104}{2}}$	4.75%

Although Case 1 and 3 increase, Case 2 decreases, and Case 4 remains constant while more decks are added to the game, neither the casino nor the player will gain an excess advantage over the other. This poses the question: "Why would casinos use a higher number of decks when their chances of winning do not vary much between decks?" The answer is in the Worst Case to Best Case Scenario Intervals.

[19] Lucas, A. F., & Spilde, K. (2019). How Changes in the House Advantages of Reel Slots Affect Game Performance. *Cornell Hospitality Quarterly*, *60*(2), 135-149.

Weighted Compressed Sample Space

The value of each matrix entry indicates the number of sample points for the corresponding matrix pair in C.S.S.

$$\text{W.C.S.S} = \sum_{ij} \left\{ \begin{matrix} 12 & 16 & 16 & 16 & 16 & 12 & 16 & 8 & 64 & 16 \\ 16 & 12 & 16 & 16 & 16 & 12 & 16 & 8 & 64 & 16 \\ 16 & 16 & 12 & 16 & 16 & 12 & 16 & 8 & 64 & 16 \\ 16 & 16 & 16 & 12 & 16 & 12 & 16 & 8 & 64 & 16 \\ 16 & 16 & 16 & 16 & 12 & 12 & 16 & 8 & 64 & 16 \\ 12 & 12 & 12 & 12 & 12 & 6 & 12 & 6 & 48 & 12 \\ 16 & 16 & 16 & 16 & 16 & 12 & 12 & 8 & 64 & 16 \\ 8 & 8 & 8 & 8 & 8 & 6 & 8 & 2 & 32 & 8 \end{matrix} \right\} w_{ij} = 1392 = 29 \times 48 \text{ simple events}$$

For the first draw

$$\underset{\text{PUSH}}{\begin{bmatrix} & & & & & & & 8 \\ & & & & & & 16 & \\ & & & & & 12 & & \\ & & & & 16 & & & \\ & & & 16 & & & & \\ & & 12 & & & & & \\ & 16 & & & & & & \\ 8 & & & & & & & \end{bmatrix}} \quad \underset{\text{DEFER}}{\begin{bmatrix} 12 & 16 & 16 & 16 & 16 & 12 & \\ 16 & 12 & 16 & 16 & 16 & & \\ 16 & 16 & 12 & 16 & & & 16 \\ 16 & 16 & 16 & & & & 16 \\ 16 & 16 & & & & & 16 \\ 12 & & & & & & 12 \\ & & & & & & 16 \\ & & & & & & \end{bmatrix}}$$

$P(\text{Push2}|\text{Defer1}) = \frac{104}{29\times48} = \frac{13}{29\times26} = 0.0747$

$P(\text{Defer2}|\text{Defer1}) = \frac{392}{29\times48} = \frac{8\times49}{29\times48} = \frac{49}{29\times6} = 0.2816$

$P(\text{BothDefer}) = \frac{29}{49} \times \frac{49}{29\times6} = 0.1667$ (Chop factor)

$P(\text{W1 or W2 or WL}) = P(\text{W1}) + P(\text{Not(T1)})\, P(\text{W2}|\text{Not(T1)}) + P(\text{Not(T1 or T2)})\, P(\text{WL}|\text{Not(T1 or T2)})$.[20]

The chop factor is the multiplier of the probability correction that would be lost if the analysis were stopped with a present dealer draw. Not much could be gained by continuing the calculation, provided the chop factor is small enough that the accompanying probability increment *P*(WL|No Early Termination) has minimal effect.

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[20] Blanchard, H. (2019). Blackjack: the math behind the cards.

Here, the chop factor implies the next correction gets chopped over 83% since *P*(No Early Termination) = 0.1667.

DEALER WIN

$$\begin{bmatrix} & & & & & & & 64 & 16 \\ & & & & & & 8 & 64 & 16 \\ & & & & & 16 & 8 & 64 & \\ & & & & 12 & 16 & 8 & & \\ & & & 12 & 12 & 16 & & & \\ & & 12 & 12 & 6 & & & & \\ & 16 & 16 & 16 & & & & & \\ 8 & 8 & 8 & & & & & & \end{bmatrix}$$

Bust or (Lose Without Bust)

$$\begin{bmatrix} & & & & & & 16 & & & \\ & & & & & 12 & & & & \\ & & B & & 16 & & & & & \\ & & & 12 & & & & & 64 & \\ & & 16 & & & & L & 8 & 64 & \\ & 12 & & & & & 12 & 6 & 48 & \\ 16 & & & & & 12 & 12 & 8 & 64 & \\ & & & & 8 & 6 & 8 & 2 & 32 & 8 \end{bmatrix}$$

P(Win2|Defer1)= $\frac{434}{29\times48}$ = $\frac{2\times7\times31}{29\times48}$ = $\frac{7\times31}{29\times24}$ = 0.31178 = *P*(DealerWins2|Defer1)

P(Lose2|Defer1) = $\frac{462}{29\times48}$ = $\frac{2\times7\times3\times11}{29\times3\times16}$ = $\frac{7\times11}{29\times8}$ =0.33146=*P*(PlayerWins2|Defer1)

P(Defer1 ∩ Win2) = $\frac{29}{49}$ × $\frac{7\times31}{29\times24}$ = $\frac{31}{7\times24}$ = 0.1845 = *P*(Dealer wins on 1st draw).

Overall outlook

P(Player Wins Before or On 1st Draw) = *P*(Dealer Loses On H.Card or 1st Draw)

P(Dealer Loses H. or 1st Draw) = *P*(Dealer Loses H.) + *P*(Defer1) × *P*(Lose2|Defer1)

P(Player Wins On or Before 1st Draw) = $\frac{16}{49}$ + $\frac{29}{49}$ × $\frac{7\times11}{29\times8}$ = 0.3265+0.1964 = 0.5229!

P(Dealer Wins On or Before 1st Draw) = 0 + 0.1845 = 0.1845 (Assume unit bets)

Players Expectation (To Win On or Before First Draw) = 0.5229 – 0.1845 = 0.3384![21]

[21] Blanchard, H. (2019). Blackjack: the math behind the cards.

Conclusions

Casino gaming is one of the most regulated industries in the world. Most gaming regulatory systems share common objectives: keep the games fair and honest and assure that players are paid if they win. Fairness and honesty are different concepts. A casino can be honest but not fair. Honesty refers to whether the casino offers games whose chance elements are random.

Fairness refers to the game advantage—how much of each dollar wagered should the casino be able to keep? A slot machine that holds, on average, 90% of every dollar bet is certainly not fair, but could very well be honest (if the outcomes of each play are not predetermined in the casino's favor). Two major regulatory issues relating to fairness and honesty—ensuring random outcomes and controlling the house advantage—are inextricably tied to mathematics and most regulatory bodies require some type of mathematical analysis to demonstrate game advantage and/or confirm that games outcomes are random.

Such evidence can range from straightforward probability analyses to computer simulations and complex statistical studies. Requirements vary across jurisdictions, but it is not uncommon to see technical language in gaming regulations concerning specific statistical tests that must be performed, confidence limits that must be met, and other mathematical specifications and standards relating to game outcomes.

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